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AVERAGE AND PROBABILITY.

172. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

A circular arc, with center at one corner of a given square, is drawn through a point taken at random in the square. What is the average length of the arc within the square?

II. Solution by HENRY HEATON, Belfield, N. D.

Let r=distance of the random point from the corner and n=number of points in the unit area. If $r<a$, the length of the arc is $\frac{1}{2}\pi r$, and the number of points from r to $r+dr$ is $\frac{1}{2}n\pi rdr$. Hence, the number of arcs of length $\frac{1}{2}\pi r$ is $\frac{1}{2}\pi nrdr$. If $r>a$ and $<a\sqrt{2}$ the length of the arc is $r[\frac{1}{2}\pi-2\cos^{-1}(a/r)]$ and the number of arcs of this length is $nr[\frac{1}{2}\pi-2\cos^{-1}(a/r)]$. To find the average, we must multiply the length of each arc by the number of times it is drawn, take the sum of the products, and divide the result by the whole number of arcs.

Hence, the required average is

$$\Delta=\{n\int_0^a \tfrac{1}{4}\pi^2 r^2 dr+n\int_a^{a\sqrt{2}} r^2[\tfrac{1}{2}\pi-2\cos^{-1}(a/r)]^2\}\div na^2$$

$$=\frac{\pi^2}{4a^2}\int_0^{a\sqrt{2}} r^2 dr+\frac{1}{a^2}\int_a^{a\sqrt{2}} 4\{[\cos^{-1}(a/r)]^2-2\pi\cos^{-1}(a/r)\}r^2 dr$$

$$=\tfrac{2}{3}a\int_0^{\frac{1}{4}\pi}(\pi-4\theta)\sec^3\theta d\theta,\text{ where }\theta=\cos^{-1}(a/r)$$

$$=\tfrac{4}{3}a[\sqrt{2}-1+\int_0^{\frac{1}{4}\pi}\log(\tan\theta+\sec\theta)d\theta]=(1.03+)a.$$

NOTE. Since the publication of our solution of this problem in the October number of the MONTHLY, we received a criticism from Mr. Heaton, to the effect that the published solution is wrong. Mr. Sanders also agrees with Mr. Heaton in his contention. Of course we expected that criticism would follow, and do not now hope to forever settle the contention that arises when problems of this sort are proposed in the *indefinite* form. However, we again repeat what we have several times said before, that when no law of distribution is assigned there are as many ways of solving the problem as there are ways of assigning laws of distribution. In such cases, there is no such thing as *the* correct solution. The above problem is stated in the indefinite form, and so one solution assuming one law of distribution is as good as another assuming some other law of distribution. We have published Mr. Heaton's elegant solution for the benefit of those who have interpreted the problem in his way. Ed. F.

177 (Incorrectly numbered 176). Proposed by T. N. HAUN, Mohawk, Tenn.

A cube being cut at random by a plane, what is the chance that the section is a hexagon? (Problem 72, p. 503, Williamson's *Integral Calculus*.)

A solution of this problem by Dr. Neikirk is given on pp. 180-182, Vol. VI, of MONTHLY. If any one has a different solution to offer, we shall be glad to consider it.